REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 82-0676	2. GOVT ACCESSION NO. AD-A118611	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) SIDELOBE MODULATION IN FAR-FIELD RADIATION PATTERNS		5. TYPE OF REPORT & PERIOD COVERED FINAL June 1981 - June 1982
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Gene Moriarty		8. CONTRACT OR GRANT NUMBER(s) AFOSR 81-0176
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Electrical Engineering School of Engineering, San Jose State Univ. San Jose, CA 95192		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2305/D9
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research Washington, D.C.		12. REPORT DATE June 1, 1982
		13. NUMBER OF PAGES 23
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release;
distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

PHASED ARRAYS, MODULATION, SIDELOBES

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this research, the following problem is addressed: How can
low sidelobes in the distant electric field or power pattern be achieved
via time-modulation of the element values in a linear phased array?
Time-modulation of the N array element values can result in far-field
patterns whose sidelobes are severely modulated. This procedure can
produce sidelobes which on the average are very low. In this research,
ON/OFF switchings are programmed in a predetermined sequence to produce

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20. a specified far-field pattern with equal and ultra-low sidelobe levels. In addition, the mainbeam of the far-field pattern is kept fairly fixed. A FORTRAN program is developed to produce ON and OFF times for this modulation scheme, given a specific value of array length (N) and a desired sidelobe level (PK).

SIDELOBE MODULATION IN FAR-FIELD RADIATION PATTERNS

by

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June 1, 1982

Technical Report No. 82-01

Prepared Under

Air Force Office of Scientific Research

Grant AFOSR 81-0176

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ACKNOWLEDGMENTS

I. INTRODUCTION

The use of time as a parameter in the design of antennas is not in itself a novel idea. The possible improvement in antenna performance via various "time-modulation" schemes was suggested more than twenty years ago [1,2]. The results of these early studies, however, did not warrent a continued research effort and the problem was bracketed. A new interest in time-modulation is developing today in large measure due to the speed and sophistication of modern digital signal processing techniques.

Linear phased array antennas, formed by a combination of individual radiating elements, have a distinct advantage over antennas of the lens or reflector type: the radar beam can be steered without mechanically moving the entire antenna structure; it is only necessary to vary electronically the relative phase between the radiating elements. This advantage increases as the size of the antenna increases. Array antennas are emphasized in this research and their time-modulation will be the primary focus of this effort.

Array antennas are characterized by the geometric position of the individual radiating elements and by the amplitude and phase of their excitation. The excitation distribution is related to the far-field radiation pattern by the Fourier Transform. Among the far-field parameters of interest in radar design are beamwidth, directivity, power gain, impedance, and sidelobe level. Low sidelobes, for example, are generally desired in most radar designs. The time-modulation scheme investigated in this research will yield parameters - which will actually be ON/OFF switching times - and the employment of these parameters will result in a far-field pattern which on the average has ultra-low sidelobes.

It is well known that a _uniform_ radar current distribution produces a far-field pattern whose first sidelobe is 13.5 dB down from the mainbeam. To achieve lower sidelobe levels, many clever schemes have been devised [3,4]. Most of these employ a _nonuniform_ current distribution across the antenna's radiating elements. The binomial distribution, for example, produces zero sidelobes, but at the expense of a very wide mainbeam. The Dolph-Chebyshev current distribution yields a far-field pattern which has minimum beamwidth for a specified sidelobe level. In theory, these distributions can produce zero or very low sidelobes. In practice, however, for a single fixed array antenna, a sidelobe level of around 40 dB is minimum. This is because very low sidelobe levels require very large current ratios across the aperture distribution and these large current ratios are impossible to maintain with the required precision using the presently available construction technologies.

Now, if the excitation distribution of an array antenna is time-modulated, then the far-field radiation or power pattern will also be time-modulated. By proper choice of a modulation function, a far-field pattern can be produced which has a fairly steady mainbeam and an effectively ultra-low sidelobe pattern. The modulation function used here is a simple ON/OFF timing scheme. The array elements are kept at a constant current value. Only the ON and the OFF times of the elements are varied.

II. OBJECTIVES

The objectives of this project were:

A. To develop a modulation scheme which employes an ON/OFF timing schedule.

B. To write a FORTRAM program that designs the proper ON/OFF schedule for a given array length and a desired far-field sidelobe level.

    C. To generate some examples to illustrate this ON/OFF approach

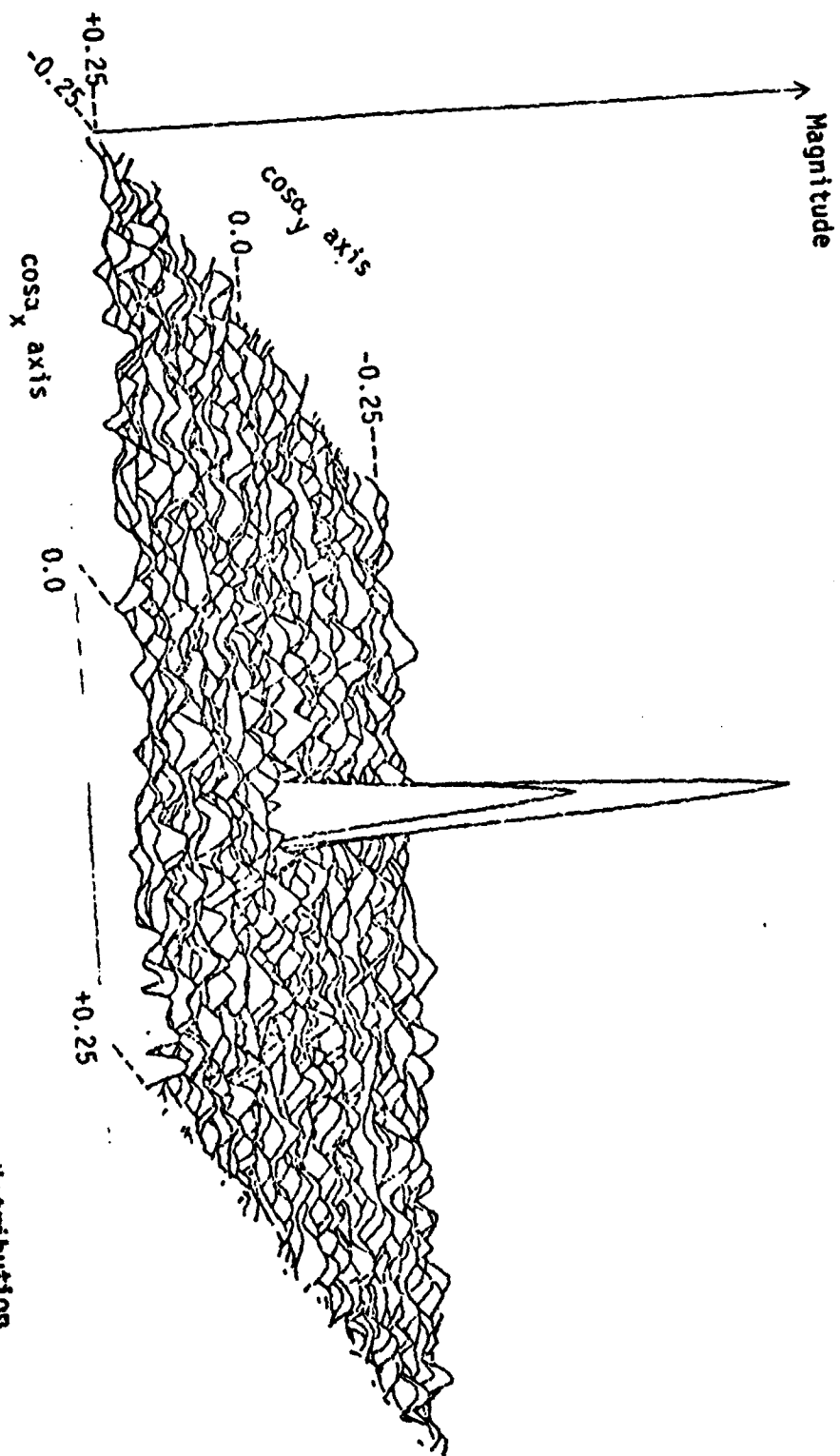   to sidelobe modulation.

III. MOTIVATION

A planar array antenna produces a three-dimensional far-field pattern. An RADC programming package is available to generate far-field plots for a wide variety of aperture current distributions [5]. For a certain current distribution, called the Taylor distribution, which is spatially thinned to yield 30 dB sidelobes, the far-field pattern appears as in Figure 1.

Now, if this spatially thinned antenna were rotated at a fixed angular rate, then the pattern of Figure 1 would rotate as well. Consider a particular point off the mainbeam, say at (θ_0, ϕ_0), in the rotating far-field. Over a period of time, T, a time signal could be observed at (θ_0, ϕ_0). The average value of this waveform would be much lower than any of the particular sidelobes indicated in Figure 1.

However, the prospect of rotating a large planar array of, for example, one thousand elements is not attractive. How can a similar result occur without needing to rotate a large antenna? Answer: by time-modulating the antenna elements in such a way that in the far-field at (θ_0, ϕ_0) a time signal appears which is similar to the time signal mentioned above. What kind of time-modulation to employ is the question addressed by this research effort.

Further motivation is provided by a consideration of the advantages of low sidelobe levels. A particular case of interest is the situation in which a jammer is present. How can a jammer be prevented from detecting the presence of the radar signal? Of course, this is impossible to do if the jammer is right in the mainbeam. However, if the jammer is at an

Figure 1   Far-field of 30 dB statistically thinned Taylor distribution.

$\cos\alpha_y = \sin\theta \, \sin\phi$
$\cos\alpha_x = \sin\theta \, \cos\phi$
θ = elevation angle
ϕ = azimuth angle

angle sufficiently away from the mainbeam, then he will see only the sidelobes.
If the sidelobes are severely modulated, then the jammer will not be able
to distinguish between the signal he detects and his own receiver noise.

The jammer problem is currently handled by adaptive radar techniques.
The presence of the jammer must first be determined at a specific (θ_o, ϕ_o).
Then, via an adaptive feedback loop, a notch is placed in the far-field
pattern at (θ_o, ϕ_o). Since the mechanism for this process is rather involved,
it is anticipated that sidelobe modulation techniques might serve as an
alternative to adaptive processing. An obvious advantage of sidelobe
modulation is that the presence of the jammer at a specific angle need not
be determined, eliminating the need for a feedback loop.

IV. ARRAY POLYNOMIALS AND FAR-FIELD EQUATIONS

Since the distribution of current across the face of a linear phased
array is related to the distant electric field by the Fourier Transform,
a uniform excitation across an array consisting of a large number of
elements yields in the distant field the familiar sin(x)/x pattern which
has a first sidelobe level of 13.5 dB down from the mainbeam. With
nonuniform excitations, the equation describing the current distribution –
called the array polynomial equation – consists of both real and imaginary
terms and is not in a very convenient form for design purposes. There
is an alternative formulation, developed by Cheng and Ma [6,7,8], which
treats a linear array of discrete elements as a sampled-data system and
employs the theory of finite Z transforms. This approach is particularly
useful for the purposes of this research and is extensively employed.

Let N be the number of elements in the linear phased array. In order
to have symmetry with respect to the center of the array, N will be restricted

to odd numbers. Only broadside arrays will be considered. The distance, d, between any two adjacent elements will be assumed to be equal. From [8] the far-field power equation is as follows:

$$P(y) = \prod_{i=1}^{(N-1)/2} (y + c_i)^2, \tag{1}$$

and the array polynomial equation is as follows:

$$E(z) = \prod_{i=1}^{(N-1)/2} (1 + c_i z^{-1} + z^{-2}). \tag{2}$$

In these equations, P varies with y and E varies with z and z and y are related by the equation: $y = z + z^{-1}$. Now, y is a cosine function given by $2 \cos\left[\frac{2\pi d}{\lambda} (\cos\theta - \cos\theta_0)\right]$, where λ is the free-space wavelength of the radiating signal, θ is the angle between the line of the linear array and a line through the center of the array, and θ_0 is the position of the principal maximum. The c_i terms which appear in both equations are the terms which constitute the weights of the array elements.

Expanding the array polynomial E(z) yields a polynomial in z^{-1} where the coefficients of z^{-1} are the array weights, symmetrical with respect to the center of the array. For example, for a five element array: $E(z) = 1 + (c_1 + c_2)z^{-1} + (2 + c_1 c_2)z^{-2} + (c_1 + c_2)z^{-3} + z^{-4}$. From this expansion, the relative weights are 1, $c_1 + c_2$, $2 + c_1 c_2$, $c_1 + c_2$, 1. If a _uniform_ array is desired, then these relative weights must be equal, resulting in values: $c_1 = -0.618$ and $c_2 = 1.618$. The power equation for this case becomes $P(y) = (y - 0.618)^2 (y + 1.618)^2$ and this equation has sidelobes maximum at y positions given by $\frac{d}{dy} P(y) = 0$. The first sidelobe

is found to be 12 dB down from the mainbeam. (This value would approach 13.5 dB as N got very large.)

V. ARRAYS WITH EQUAL SIDELOBES

Given a set of c_i terms, a certain array polynomial E(z) results and this array yields a certain power equation P(y). P(y) can be plotted vs. y and the far-field characteristics can be studied. This is the analysis problem. The design problem requires the designer to arrive at a certain set of c_i terms which meet some specific requirements usually expressed in terms of the far-field characteristics. For example, given the requirement that the far-field has equal sidelobes and these levels are all to be at a specific dB value down from the mainbeam, determine the particular set of c_i terms which meets these specs.

In [8] a technique for designing arrays with equal sidelobes is presented. The procedure is based on manipulating the power pattern equation P(y). There are three steps involved:

1. Solve for the positions, y_s, of all the sidelobes from $\frac{d}{dy} P(y) = 0$.

2. Equate all $P(y_s)$ terms.

3. Set the ratio $P(2)/P(y_s)$ equal to some constant, say k^2, which is fixed by a desired sidelobe level relative to the principal maximum.

For N an odd number, there are (N-1)/2 distinct values of c_i to be determined. In [8] there is a general and simple relationship developed between the c_i terms and the y_s terms. This relationship is derived in terms of the known properties associated with a Chebyshov polynomial. These polynomials have the interesting property of displaying equal ripples which, in the case at hand, are sidelobes of the P(y) polynomial.

The equations to be solved from [8] are as follows:

$$2 - c_1 = (2 - c_i) \left[\frac{\cos^2 \frac{\pi}{2} (N - 1)}{\cos^2 \frac{\pi}{2} \frac{(2i-1)}{(N-1)}} \right] \qquad \text{for } i = 2, \ldots, \frac{N-1}{2} \qquad (3)$$

and

$$2 - c_1 = (2 + y_i) \left[\frac{\cos^2 \frac{\pi}{2} (N-1)}{\cos^2 \frac{\pi i}{N-1}} \right] \qquad \text{for } i = 1, \ldots, \frac{N-3}{2} \qquad (4)$$

and $\quad \dfrac{P(2)}{P(y_s)} = k^2 \qquad\qquad\qquad\qquad\qquad\qquad (5)$

Since $y = -2$ is always a sidelobe, it is convenient to express equation (5) as $P(2)/P(-2) = k^2$. If -20 dB sidelobes are desired, let $k^2 = 100$ or $k = 10$. So, $20 \log k = 20 \log 10 = 20$ dB. If -80 dB sidelobes are desired, let $k^2 = 10^8$ or $k = 10^4$. In this case, $20 \log k = 80$ dB. Note that even though (3) and (4) are linear equations, since (5) is nonlinear and (3), (4), and (5) need to be solved simultaneously, there is a need for a nonlinear equation solving routine. The IMSL subroutine ZSYSTM solves N simultaneous nonlinear equations in N unknowns and has proven very effective in this research.

As an example, an eleven-element array with equal sidelobes would yield a power equation $P(y)$ which, when plotted vs. y, appears as in Figure 2. The far-field pattern for these broadside arrays has a visible range which extends to the left of $y = 2$ and is symmetric with respect to the vertical axis for the standard case of $d = \lambda/2$. Note that zeros of $P(y)$ occur at c_i values and sidelobes peak at y_s values. Once the c_i terms are known, the actual array weights are determined by expanding $E(z)$. These coefficients
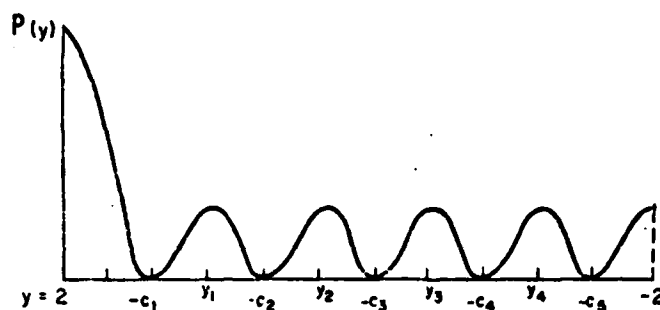
Figure 2   Relative positions of nulls and sidelobes for an eleven-element array with equal sidelobes.

of Z^{-i} in the E(z) expansion are the normalized array weights, which will be called A(i), i = 1, ..., N.

This so far is one standard design approach to linear phased arrays. If an eleven-element array is designed with sidelobes of -80 dB, the power pattern would in theory appear as in Figure 2. However, the problem – as mentioned initially – is that the precision required in actually constructing such an array is impossible to achieve. In actual practice, the sidelobes produced as in Figure 2 might be 30 or 40 dB down from the mainbeam, even though according to theory they should be 80 dB down. A designer's dilemma? Time-modulation is one way out of this dilemma.

VI.   THE ON/OFF TIME-MODULATION SCHEME

In the work of Kummer, et al. [2] from 1963, the ON/OFF time-modulation problem was addressed. Their approach, however, was not very systematic. They turned array elements ON and OFF over a fixed time period and demonstrated the fact that the far-field patterns in response to the modulation did yield reduced average sidelobe levels. They did not present a systematic design

approach nor did they attempt to keep the mainbeam fixed throughout the modulation period.

The current research effort took off from where [2] left off. The problem considered was this: How can a specified far-field pattern be produced which has sidelobes that are severely modulated and average out to be some specific ultra-low value? In addition, it was required to keep the mainbeam relatively fixed throughout the modulation period.

The ON/OFF modulation design scheme proposed as a result of this research effort will be introduced in terms of a simple example. Assume there exists a fifteen-element linear phased array which must produce equal sidelobes 20 dB down from the mainbeam. How should it be designed? Letting $K^2 = 100$ and $N = 15$ in equations (3), (4), and (5), the proper c_i terms are computed and when entered into the $E(z)$ equation (2), the resulting normalized array weights, $A(i)$, $i = 1, \ldots, 15$ appear as in Figure 3.
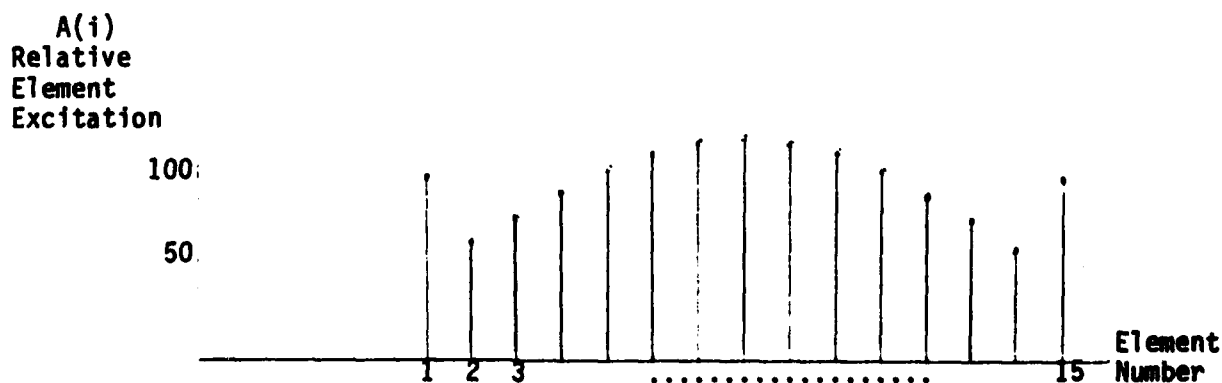

Figure 3 Relative weights for a fifteen-element array producing 20dB sidelobes.

Note that this distribution is not very different from the uniform distribution which yields a first sidelobe level of 13.5 dB down from the mainbeam. To produce the same 20 dB sidelobes which the array represented in Figure 3 produces, the modulation scheme proposed in this research requires the following:

1. Let each of the fifteen elements carry exactly the same excitation, for example, $A(i) = 1$ for all i.

2. Perform the modulation by adjusting the ON times and the OFF times for each element in such a manner that the total amount of time a given element is ON during one period of the modulation is proportional to its relative weight.

For example, element no. 2 has an initial relative weight of $A(2) = 62.0936$. Its ON time is set at 0.0000. Its OFF time is set at 62.0936. Since these times are relative, hardware limitations will dictate the actual units. Generally, it is best to have the overall modulation time as short as possible. The single key idea in the ON/OFF modulation scheme is this: the precision required in the element weights is traded for a precision required in setting ON and OFF times. The advantages of this scheme should prove to be considerable in view of the ever increasing speed of integrated circuit switches.

In Figure 4 is plotted the modulation timing scheme which employs elements of constant excitation ($A(i) =$ constant for all i) and produces in the far-field a pattern with 20 dB sidelobes. This staggering is essential in order that the same number of elements are ON "most of the time" during the modulation period. The modulation period, T, in this case is 221.3724. The ideal situation would be to keep exactly $(N-1)/2$ elements ON at all times, i.e., seven elements for this example. Keeping this number fixed over the entire modulation period would guarantee that the mainbeam stayed fixed at all times. There are some brief intervals toward the middle of the modulation

period during which there are eight elements ON or six elements ON. This will tend to produce a slight flicker in the mainbeam of the radar's far-field radiation pattern. This should not be very significant, however, since there are exactly seven elements ON for approximately 95% of the modulation period.
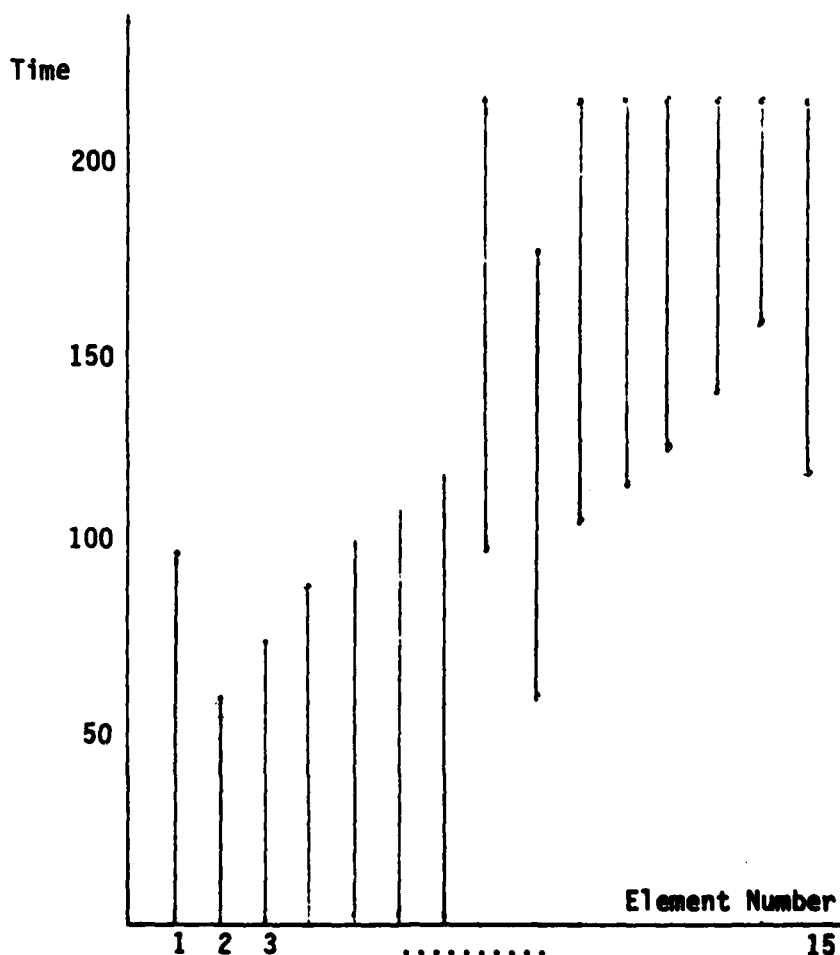

Figure 4 Timing of fifteen elements of equal excitation modulated for 20dB sidelobes.

The details of the staggered scheme exemplified in Figure 4 are presented next.

A(i)
Relative
Element
Excitation

12000

10000

8000

6000

4000

2000

1   2   3   15   Element
                                                                Number
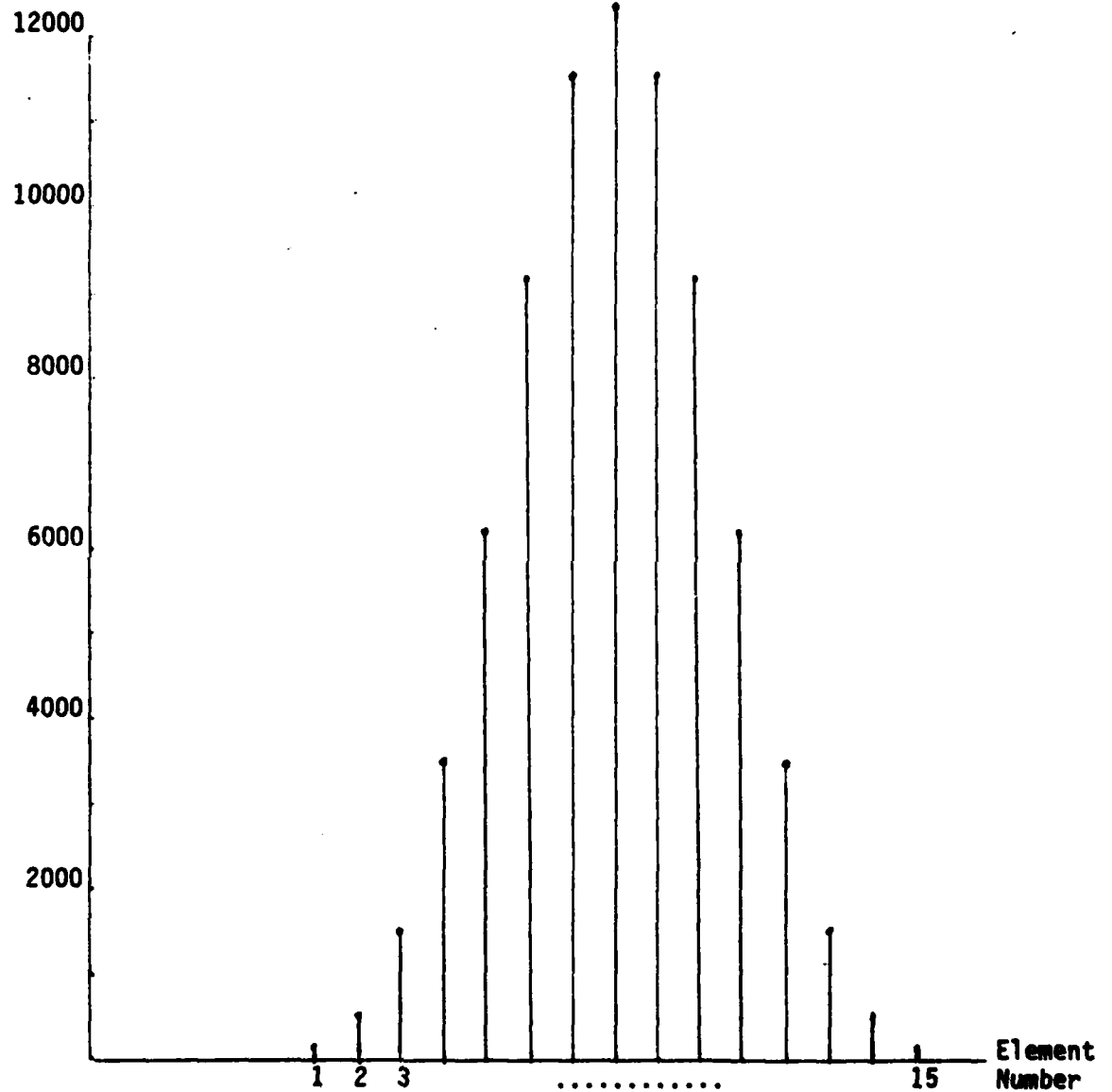
Figure 5   Relative weights for a fifteen-element array
            producing 80dB sidelobes.

required to produce sidelobes of 80 dB down from the mainbeam. This is quite a radical difference from the taper of the 20 dB case in Figure 3. However, with the time-modulation scheme proposed in this research, each element carries exactly the same excitation, say $A(i) = 1$, for all i. Again, the ON and OFF times are adjusted such that the total ON time per period for each element is proportional to the weight it is required to carry if there were no modulation. The timing schedule for this case is plotted in Figure 6.

As another example, let $N = 21$ and $PK = 80$ (21 element array designed to produce equal sidelobes 80 dB down from the mainbeam). The ON and OFF times for each element are presented in Table 1.

Table 1. ON and OFF times for a twenty-one-element array modulated to produce 80 dB sidelobes.

ELEMENT NUMBER	ON TIME t_{ON}	OFF TIME t_{OFF}
1	0	100.0000
2	0	420.0948
3	0	1170.1288
4	0	2573.2972
5	0	4794.4197
6	0	7844.8256
7	0	11511.8494
8	0	15350.5167
9	0	18757.2815
10	0	21113.7267
11	100.0000	22056.4860
12	420.0948	21533.8215
13	3299.2045	22056.4860
14	6705.9693	22056.4860
15	10544.6366	22056.4860
16	14211.6604	22056.4860
17	17262.0663	22056.4860
18	19483.1388	22056.4860
19	20886.3572	22056.4860
20	21636.3912	22056.4860
21	21956.4860	22056.4860
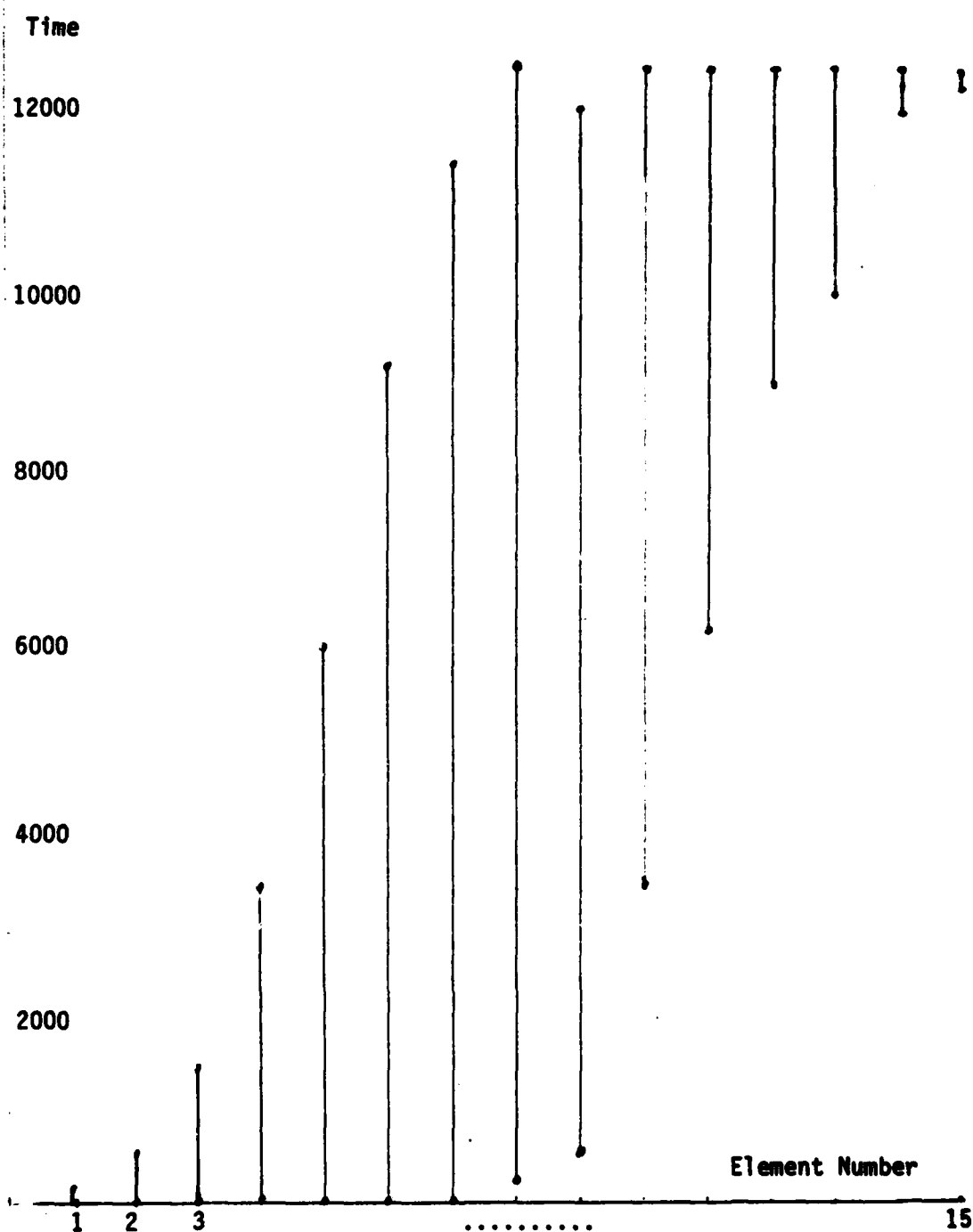
Figure 6   Timing of fifteen elements of equal excitation modulated for 80dB sidelobes.

A final example: N = 35, PK = 80. Results are presented in Table 2.

Table 2. ON and OFF times for a thirty-five-element array modulated to produce 80 dB sidelobes.

ELEMENT NUMBER	ON TIME t_{ON}	OFF TIME t_{OFF}
1	0	100.0000
2	0	272.8930
3	0	612.3810
4	0	1184.8680
5	0	2068.2561
6	0	3337.0125
7	0	5050.7823
8	0	7242.1780
9	0	9905.5033
10	0	12988.4345
11	0	16388.4301
12	0	19955.1987
13	0	23499.6751
14	0	26809.0369
15	0	29666.2439
16	0	31871.8106
17	0	33264.9762
18	100.0000	33841.4143
19	272.8930	33537.8692
20	1969.6036	33841.4143
21	4175.1704	33841.4143
22	7032.3774	33841.4143
23	10341.7392	33841.4143
24	13886.2156	33841.4143
25	17452.9842	33841.4143
26	20852.9798	33841.4143
27	23935.9109	33841.4143
28	26599.2363	33841.4143
29	28790.6320	33841.4143
30	30504.4017	33841.4143
31	31773.1582	33841.4143
32	32656.5463	33841.4143
33	33229.0332	33841.4143
34	33568.5213	33841.4143
35	33741.4143	33841.4143

IX. CONCLUSIONS AND RECOMMENDATIONS

A method of sidelobe modulation involving an ON/OFF timing scheme
has been presented. The method relies on precision in setting ON and OFF
times. To the question: How much precision?, the only answer is: As
much as possible. The point is that timing precision is greater than the
precision available to set array element excitation weights. Actual
numbers for comparison purposes are difficult to come up with here.
Nevertheless, the implementation of some of these timing schedules should
answer some questions and is an anxiously awaited future endeavor.

It is anticipated that the far-field patterns designed to have, say,
80 dB sidelobes will have perhaps 70 or 60 dB sidelobe levels. It is
inevitable that no system is 100% efficient. Some problems here are that
the ON/OFF times will never all be able to be exactly set. This is the
precision problem. The elements will never all be able to carry exactly
the same excitation: There may be fixed biases on the amplitude and/or
phase of some particular elements. The spacings between array elements
can never be made exactly equal. In addition, most systems have noise
problems. To be sure, from a variety of sources, there will be system
degradation. Nevertheless, the future of ON/OFF modulation looks bright,
especially in view of the ever increasing efficiency of integrated circuit
technology.

One further aspect of this research that might bear additional con-
sideration is the method of staggering the ON/OFF times. The problem of
exactly where to place the t_{OFF} value for the $(\frac{N+5}{2})$th to the Nth element
has been solved in a rather empirical fashion from examination of a large
number of cases. There might be a more optimal approach to this problem.
Perhaps t_{OFF} (N) could be chosen in such a way that the modulation period,

T, is minimized; or, t_{OFF} (N) could be chosen such that the percentage of time during which there are exactly $\frac{N-1}{2}$ elements ON is maximized.

With regard to the possible imprecision in time settings, an interesting recommended future study would be to consider a sensitivity analysis. An analysis of sensitivities of c_i parameters has been presented in [7]. Perhaps this could be extended to the case of the t_{ON} and t_{OFF} parameters.

Some final recommendations are to study the extention of this ON/OFF approach to sidelobe modulation to the case of planer arrays, to consider the advantages of non-equally spaced array elements in linear arrays, and to explore not just modulation of the amplitudes of the array elements but modulation of both amplitudes and phases.

APPENDIX

```
      PROGRAM SDLB(INPUT,OUTPUT,TAPE6=OUTPUT)
C
C     THIS PROGRAM PRODUCES A LINEAR PHASED-ARRAY DESIGN SCHEME WHICH
C     HAS A FAR-FIELD RADIATION PATTERN WITH EQUAL AND ULTRA-LOW
C     SIDELOBES. THE NUMBER - N -  OF EQUALLY SPACED IDENTICAL
C     ELEMENTS MUST BE ODD. THE ONLY INPUTS REQUIRED ARE N AND
C     THE DESIRED SIDELOBE LEVEL, PK.
C
      EXTERNAL AUX
      REAL PAR(4),WA(5453),X(101),C(50),AA(101),FST(101),SND(101)
      REAL TON(101),TOFF(101)
C
C     THE NEXT STATEMENTS ARE THE INPUTS: N AND PK
C

      N=35
      PK=80.

      PI=3.1416
      PAR(1)=PI
      PAR(2)=10.0**(PK/20.0)
      PAR(3)=COS(PI*0.5/(N-1))**2
      PAR(4)=N
      EPS=0.01
      ITMAX=100
      NSIG=3
      DO 15 I=1,N
      X(I)=0.0
   15 CONTINUE
      NNN=N-2
C
C     THE SUBROUTINE CALLED NEXT - ZSYSTM - WILL SOLVE THE NONLINEAR
C     ALGEBRAIC EQUATIONS REQUIRED TO PRODUCE THE PROPER ARRAY
C     WEIGHTS WHICH YIELD THE FAR-FIELD SIDELOBE LEVELS, PK.
C
      CALL ZSYSTM(AUX,EPS,NSIG,NNN,X,ITMAX,WA,PAR,IER)
      WRITE(6,20) (X(I),I=1,N)
   20 FORMAT(10F10.4)
      M=(N-1)/2
      DO 25 I=1,M
      C(I)=X(I)
   25 CONTINUE
      DO 30 J=1,N
      FST(J)=0.0
      SND(J)=0.0
   30 CONTINUE
```

```
      FST(1)=1.0
      FST(2)=C(1)
      FST(3)=1.0
      DO 35 J=2,M
      DO 40 I=1,N
      SND(I)=FST(I)
   40 CONTINUE
      NN=N-1
      DO 45 I=1,NN
      SND(I+1)=SND(I+1)+C(J)*FST(I)
   45 CONTINUE
      DO 50 I=1,NNN
      SND(I+2)=SND(I+2)+FST(I)
   50 CONTINUE
      DO 55 I=1,N
      FST(I)=SND(I)
   55 CONTINUE
   35 CONTINUE
      DO 60 I=1,N
      AA(I)=FST(I)
      AA(I)=100.0*AA(I)
   60 CONTINUE
C
C     AT THIS POINT, THE RELATIVE WEIGHTS ARE PRINTED OUT
C
      WRITE(6,79)
   79 FORMAT(1H1,77H THE FOLLOWING ARE THE RELATIVE WEIGHTS RESPECTIVELY
     1 FOR THE N ARRAY ELEMENTS)
      WRITE(6,80) (AA(I),I=1,N)
   80 FORMAT(10F13.4)
      DO 81 I=1,M
      TON(I)=0.0
   81 CONTINUE
      TOFF(1)=AA(1)
      TOFFM=TOFF(1)
      DO 82 I=2,M
      TOFF(I)=AA(I)
      IF(TOFF(I).GT.TOFFM)  TOFFM=TOFF(I)
   82 CONTINUE
      MM=(N+1)/2
      TON(MM)=TOFF(1)
      TOFF(MM)=TON(MM)+AA(MM)
      M1=MM+1
      M2=M1+1
      TON(M1)=TOFF(2)
      TOFF(M1)=TON(M1)+AA(M1)
      IF(TOFF(MM).GT.TOFFM)  TOFFM=TOFF(MM)
      IF(TOFF(M1).GT.TOFFM)  TOFFM=TOFF(M1)
      DO 83 I=M2,N
      TOFF(I)=TOFFM
   83 CONTINUE
```

```
      DO 88 I=M2,N
      TON(I)=TOFF(I)=AA(I)
   88 CONTINUE
C
C     AT THIS POINT, THE RELATIVE ON AND OFF TIMES ARE PRINTED OUT
C
      WRITE(6,89)
   89 FORMAT(/,66H THE RELATIVE TURN-ON TIMES RESPECTIVELY FOR THE N ARR
     1AY ELEMENTS:)
      WRITE(6,90) (TON(I),I=1,N)
   90 FORMAT(10F13.4)
      WRITE(6,91)
   91 FORMAT(/,67H THE RELATIVE TURN-OFF TIMES RESPECTIVELY FOR THE N AR
     1RAY ELEMENTS:)
      WRITE(6,92) (TOFF(I),I=1,N)
   92 FORMAT(10F13.4)
      STOP
      END



      REAL FUNCTION AUX(X,K,PAR)
      REAL PAR(4),WA(5453),X(161)
      N=PAR(4)
      A1=PAR(3)
      PI=PAR(1)
      M=(N-1)/2
      L=(N-3)/2
      I=K+1
      IF(I.GT.M) GO TO 20
      AUX=X(1)=2.0+(2.0-X(I))*A1/COS((2*I-1)*PI/(2*(N-1)))**2
      RETURN
   20 IF(I.GT.M+L) GO TO 30
      AUX=X(1)=2.0+(2.0+X(I))*A1/COS((I-M)*PI/(N-1))**2
      RETURN
   30 P2=1.0
      DO 40 J=1,M
      P2=P2+(2.0+X(J))**2
   40 CONTINUE
      PM2=1.0
      DO 50 J=1,M
      PM2=PM2+(-2.0+X(J))**2
   50 CONTINUE
      AUX=P2-PM2*PAR(2)**2
      RETURN
      END
```

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